I. Ratio

The relation which one quantity bears to another quantity of the same kind, showing the number of times one quantity is contained in another is called ratio between the two quantities.

The ratio a : b represents a fraction $\frac{a}{b}$

The quantity in the numerator is called antecedent and the quantity in the denominator is called consequent.

e.g. The ratio of Rs. 25 to Rs. 50 is 25 : 50 or $\frac{25}{50}$ or 1 :2

e.g. The ratio of 5 km to 600 m is 5000 : 600 or $\frac{5000}{600}$ or 25 : 3

II. Composition of Ratios

1) Compounded Ratio

The compounded ratio of a : b and c : d is ac : bd

The compounded ratio of a : b, c : d and e : f is ace : bdf

2) Duplicate Ratio

The duplicate ratio of a : b is $a^2 : b^2$

3) Triplicate Ratio

The triplicate ration of a : b is $a^3 : b^3$

4) Sub – duplicate Ratio

The sub – duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$

5) Sub – triplicate ratio

The sub – triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$

6) Inverse Ratio

The inverse ratio or reciprocal ratio of a : b is $\frac{1}{a} : \frac{1}{b}$ => b : a

III. Comparison of Ratios

Lat a : b and c : d be two ratios, then

(i) a : b > c : d if ad > bc

(ii) a : b < c : d if ad < bc

(iii) a : b = c : d if ad = bc

IV. Ratio of greater inequality or lesser inequality or equality

1) A ratio a : b is called a ratio of greater inequality if a > b

e.g. 4 : 3 is the ratio of greater inequality

A ratio of greater inequality is decreased if the same positive number is added to both the terms of the ratio.

$\frac{4}{3} > \frac{4+2}{3+2}$ i.e. $\frac{4}{3} > \frac{6}{5}$

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A ratio of greater inequality is increased if the same positive number is subtracted from both the terms of the ratio.

$\frac{4}{3} < \frac{4-2}{3-2}$ i.e. $\frac{4}{3} < \frac{2}{1}$

2) A ratio a : b is called a ratio of less inequality if a < b

e.g. 3 : 5 is the ratio of less inequality

A ratio of less inequality is decreased if the same positive number is added to both the terms of the ratio

$\frac{3}{5} < \frac{3+2}{5+2}$ i.e. $\frac{3}{5} < \frac{5}{7}$

A ratio of less inequality is decreased if the same positive number is subtracted from both the terms of the ratio

$\frac{3}{5} > \frac{3-2}{5-2}$ i.e. $\frac{3}{5} > \frac{1}{3}$

v. Proportion

Four quantities a, b, c ,d are said to be in proportion when the ratio of first to the second is the same as the ratio of third to the fourth

i.e, a : b = c : d or a : b : : c : d

The terms a and d are called extremes and b and c are called means.

e.g. 2, 5, 12, 30 are in proportion, because 2 : 5 = 12 : 30

1) Continues proportion

Four quantities a, b, c ,d are said to be in continued proportion, when

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

2) Mean proportion (Geometric Mean)

If three quantities a, b and c are in continued proportion, then

$\frac{a}{b} = \frac{b}{c}$ => b^2 = ac

Here b is called the geometric mean of a and c

vi. Some useful results on proportion

1) Invertendo

If a : b : : c : d, then b : a : : b : c

Or $\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{b}{a} = \frac{d}{c}$

2) Alternendo

If a : b : : c : d, then a : c : : b : d

Or $\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{a}{c} = \frac{b}{d}$

3) Componendo

If a :v : : c :d, then (a + b) : b : : (c + d) : d

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Or $\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{a+b}{d} = \frac{c+d}{d}$

4) Dividendo

If a : b : : c : d, then (a – b) : : (c + d) : (c -d)

Or $\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

Some important points

1) The value of a ratio remains same, if both the antecedent and the consequent are multiplied or divided by the same non – zero quantity

$\frac{a}{b} = \frac{ma}{mb}$ and $\frac{a}{b} = \frac{a/m}{b/m}$ where m ≠ 0

2) The ratio of two fractions can be expressed as a ratio of two integers

e.g. $\frac{a}{b} : \frac{c}{d}$, then this ratio is equal to ad : bc

3) Two quantities are said to be commensurable, if their ratio can be expressed exactly by the ratio of two integers, otherwise the quantities are said to be incommensurable

e.g. The ratio of 20 m to 12 m is 20 : 12 = 5 : 3, thus 20 m and 12 m are commensurable quantities.

e.g. The ratio of π to $\sqrt{2}$ can not be expressed or ratio of two integers, so π and $\sqrt{2}$ are incommensurable quantities.

4) If four quantities are in proportion, the product of extremes is equal to the product of mean.

a : b : : c : d => product of extremes = product of mean

⇨ a x d = b x c

5) the mean proportional between any two numbers is equal to the square root of their product

b = $\sqrt{ac}$

6) the fourth proportional of a, b, c is d, if a : b = c : d

7) the third proportional of a, b is c if a : b = b : c

8) if some ratios are equal, then each of these ratios is equal to the ratio of sum of all antecedents to the sum of all consequents.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} =$ ..... Then each ratio is equal to $\frac{a+c+e+\cdots}{b+d+f+\cdots}$

9) ratio of more than two quantities can be determined by as follows

if $\frac{A}{B} = \frac{a}{b}$ and $\frac{B}{C} = \frac{c}{d}$ => A : B : C = ac : bc : bd

similarly

if $\frac{A}{B} = \frac{a}{b}, \frac{B}{C} = \frac{c}{d}$ and $\frac{C}{D} = \frac{e}{f}$ => A : B : C : D = ace : bce : bde : bdf

this type of ratio is known or Continued ratio

10) if in x liter of mixture of two quantities, the ratio of two quantities is given by a : b and the second quantity is increased by y liters to make the ratio c : d then y is given by

$y = x\frac{(ad-bc)}{(a+b)\times c}$

Practice Problems

1) The ratio between two numbers is 3 : 4 and their sum is 490. Find the numbers

a) **210, 280** b) 210, 240
c) 280, 230 d) 230, 220

2) A, B, C and D are four quantities of the same kind such that A : B = 3 : 4, B : C = 8 : 9 and C : D = 15 : 16. Find the ratio

a) 30 : 34 : 45 : 48
b) 45 : 33 : 55 : 36
c) **30 : 40 : 45 : 48**
d) 33 : 40 : 55 : 32

3) In an examination, 25 students out of 70 scored less than 50% marks. Find the ratio of number of students who scored 50% marks or more to the number of students who scored less than 50% marks.

a) 8 : 9 **b) 9 : 5**
c) 5 : 8 d) 5 : 2

4) Find the ratio of third proportional to 12 and 30 and the mean proportional of 9 and 25.

a) 2 : 9 b) 3 : 2
c) **5 : 1** d) 3 : 1

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5) What should be added to the terms of the ratio 3 : 5 as to get 2 : 3?

a) 0 b) -11
c) – 1 **d) 1**

6) In a mixture of 60 litres, the ratio of milk and water is 5 : 1. How much water should be added to make the ratio 2 : 1?

a) 12 litres b) 14 litres
c) 13 litres **d) 15 litres**

7) A and B have incomes in the ratio 5 : 3. The expenses of A, B and C are in the 8 : 5 : 2. If C spends Rs. 2000 and B saves Rs. 700. Find the saving of A.

a) 1300 **b) 1500**
c) 1400 d) 1600

8) An ornament weighs 12.5 gms of which 2.5 gm is pure silver and the rest alloy. Find the ratio of pure silver to alloy?

a) 1 : 9 b) 1 : 5
c) **1 : 4** d) 1 : 3

9) The ratio of X : Y is 7 : 9 and Y : Z is 6 : 7. The ratio of X : Z is

a) 2 : 3 b) 3 : 2
c) 1 : 2 d) 2 : 1

10) A, B, C, D are four numbers so that A : B = 2 : 3, B : C = 4 : 5, C : D = 5 : 8, then A : D is

a) 2 : 3 b) 3 : 2
c) 1 : 3 d) 3 : 1

11) If 12 : x = 15 : 3, then x is

a) $2\frac{1}{5}$ b) $2\frac{3}{5}$
c) $2\frac{4}{5}$ **d) $2\frac{2}{5}$**

12) The third proportional to 12 and 30 is

a) 40 b) 45
c) 50 **d) 75**

13) The fourth proportional to 3,4 and 15 is

a) 20 b) 18
c) 17 d) 15

14) If $x : 2\frac{1}{3} :: 21 : 50$, then the value of x is

a) $\frac{27}{50}$ **b) $\frac{49}{50}$**
c) $1\frac{1}{50}$ d) $1\frac{1}{49}$

15) If x : y = 3 : 4, then (4x + 5y) : (5x - 2y) is

a) 4 : 5 **b) 32 : 7**
c) 48 : 15 d) 10 : 21

16) If 15% of x is the same as 20% of y, then x : y is

a) 3: 4 **b) 4 :3**

c) 17 : 16 d) 16 : 17

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