PROBABILITY

I.INDRODUCTION

Probability is a term between happening of an event and non-happening of that event, e.g. in tossing a coin there are 50 percent chances to come out head and 50 percent chances to come out tail. Thus, the chance of the happening of an event is termed as probability. In other words 'Probability is a measure of uncertainty.

II. EXPERIMENT

An activity which is repeated under given condition associated to certain outcome is called an experiment.

III. RANDOM EXPERIMENT

An experiment, when repeated under identical condition does not give always same result but may result one of the several possible outcome is called a random experiment. E.g

i) Tossing a coin
ii) Throwing a die
iii) Selecting a card from the pack of card etc.

IV. SAMPLE SPACE

The set of all possible outcome of a random experiment is called Sample space. It is denoted by S.

e.g. S = (HH, HT, TH, TT)

where H refer to 'Head' of acoin.

And T refer to 'Tail" of a coin.

V. SIMPLE EVENT

Single possible outcome of an experiment is called Simple Event.

e.g. on rolling a die, the chances of coming out (3) is an example of Simple Event.

VIII. COMPOUND EVENT

Simultaneous (joint) occurrence of two or more simple event is called compound event.

e.g. in rolling a die, the chance of getting (3) is a simple event and chance of getting an odd number (1, 3, 5) is a compound event.

VIII. CERTAIN EVENT

The sample space S itself is a CERTAIN or SURE EVENT.

e.g. in roll a die, the chances of getting a number less than 7 is a certain event.

IX. IMPOSSIBLE EVENT

When there is absolutely no chance of getting the possible outcome, it is called impossible event. It correspond to null set.

X. INDEPENDENT EVENT

An event A said to be independent of another event B, if the chances of occurence of event A does not influence the chance of occurrence or non-occurrence of another event B.

e.g. in tossing a coin twice, the chances of coming out head in the first case is independent of chances of coming out head in the second case.

XI. DEPENDENT EVENT

An event A is said to be dependent of another event B, if chances of occurrence or non-occurrence of event A influence the chances of occurrence or non-occurrence of another event B. i.e. event A occurs only if event B has already occurred.

e.g. if we choose two red balls from a set of 25 coloured balls then this event is dependent as in the second case the number of available balls are less.

XII.MUTUALLY EXCLUSIVE EVENT

If two events cannot occure together, then they are called mutually exclusive event. i.e. if occurrence of one excludes yhe event.

i.e. in tossing a coin, the event 'Head' and 'Tail' are mutually exclusive as both cannot occure together in single toss.

XIII. EQUALLY LIKELY EVENT

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Two or more events are said to be equally likely if no event occurs in preference to other event i.e if each has the same chances of occurrence.
e.g. in tossing a coin, there are equal chance of coming 'Head' or 'Tail'.

XIV. EXHAUSTIVE EVENT

If an event results in all possible outcome them it is called exhaustive event.
e.g. in drawing a card from a set of 52 playing cards, there are 52 exhaustive events.

XV. FAVORABLE NUMBER OF CASES

The number of cases which favours the event is called favorable number of cases.
e.g. In a single throw of die, the event A is getting multiple of 2 is given by
A = (2, 4, 6). Therefore, there are three numbers of favourable cases.

XVI. CLASSICAL DEFINITION OF PROBABILITY OF AN EVENT

Let there are n equally likely, mutually exclusive and exhaustive simple event associated with a random experiment and m of them are favorable outcomes of the event A, then the probability of event A denoted by P(A) is given by

$$P(A) = \frac{m}{n} = \frac{Number\ of\ outcome\ favorable\ to\ A}{Total\ number\ of\ outcm}$$

Where $0 \leq P(A) \leq 1$.

XVII. AXIMATIC APPROACH TO PROBABILITY

Before proceeding further, we must clear the meaning of symbols related with set Theory.

1. P(A) = Probability of occurrence of an event A.
2. $P(A) = \bar{}$ Probability of non-occurrence of event A.
3. P(A U B) = Probability of occurrence of at least one of the event either A or B or both.
4. $P(A \cap B)$ = Probability of occurrence common to both event A and B.
5. P(AB) = Probability of occurrence of event A but non- occurrence of event B.
6. P(A/B) = Probability of occurrence of event A, when the event B has already occurred.

XVIII. ADDITION THEOREM OF PROBABILITY

If A and B are two events then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

XIX. MULTIPLICATION THEOREM OF PROBABILITY

If A and B are two events then

$P(A \cap B) = P(A).\ P(B/A)$

Where $P(A) > 0$

Or $P(A \cap B) = P(B).P(A/B)$

Where $P(B) > 0$

XX. LAW OF TOTAL PROBABILITY

Let $A_1, A_2 \ldots\ldots A_n$ be n muitually exclusive exhaustive events associated with random experiment and A is any event occurs with a_1 or A_2 or $A_3 \ldots$. Or A_n, then

$P(A) = P(A_1).P(A/A_1) + P(A_2).P(A/A_2) + \ldots. + P(A_n)\ .\ P(A/A_n)$

XXI. SOME USEFUL RESULTS

1. $P(\bar{A}) = 1 - P(A)$
   $= 1 - \frac{m}{n} = \frac{n-m}{n}$
   $= \bar{P}(A) = \frac{n-m}{m}$
2. $P(A) + \bar{P}(A) = 1$
3. If A and B are mutually exclusive events, then $P(A \cap B) = 0$
   $P(A \cup B) = P(A) + P(B)$
4. For three events A, B and C, addition theorem is given by
   $P(A \cup B \cap C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + p(A \cap B \cap C)$
5. If A1, A2 ..... An are n mutually exclusive events, then by addition theorem
   $P(A1 \cup A2 \cup \ldots.. \cup An) = P(A1) + P(A2) \ldots.. + P(An)$
6. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$
7. $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

8. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$
   Or
   $P(\overline{A \cap B}) = 1 - [P(A) + (B) - P(A \cap B)$
9. $P(\overline{A}/B) = 1 - P(A/B)$
10. For conditional probability i.e. when events are dependent
    i) $P(A \cup B) = P(A) + P(B) - P(A).P(B/A)$ (by multiplication theorem of probability)
    ii) $P(A \cup B) = P(A) + P(B) - P(B).P(A/B)$
11. If A and B are independent event then
    $P(A \cup B) = P(A).P(B)$
12. For independent event A and B
    $P(A \cup B) = P(A) + P(B) - P(a).P(B)$

XXII. SOME DETAILS ABOUT COMMON EVENT

1.TOSSING A COIN
On tossing a coin, either a Head (H) or a Tail (T) appears on the upper most face.

2. THROWING A DIE
A die is a cube having 6 faces on which number or dots from 1 to 6 is marked. When we throw a die, there are chances of any of the number (1,2,3,4,5,6) to come out on the upper most face.

3. DRAWING A CARD FROM A PACK OF PLAYING CARD
A pack of playing card has 52 cards in it. It has 26 black card and 26 card. Again 26 black card has 13 spades and has 13 spades and 13 club and red card has 13 hearts and 13 diamonds.
In 13 cards of each suit (i.e spades, clubs, hearts and diamonds) there are four honours namely ace, king, queen and jack. These are called face cards.

SOME IMPORTANT POINTS

1. The process of conducting experiment is called trial, whereas the outcome is called event.
2. A simple event is also called elementary event or indecomposible vent, while compound event is called decomposable event.
3. Probability of an event always lies between 0 and 1.
   i.e. $0 \leq P(A) \leq 1$
   where P(A) is the probability of any event denoted by A.
4. Probability of an event cannot be negative.
5. The probability for sure event is 1 i.e $P(A) = 1$
6. The probability for impossible event is 0
   i.e. $P(A) = 0$
7. Sometimes probability of an event is expressed in terms of 'odds'. When the ratio of the probability of the occurrence of an event A to the probability of non-occurrence is m: n then
   If odds in favor of A are m : n, then $P(A) = \frac{m}{m+n}$
   And if odds against A are m : n, then $P(A) = \frac{n}{m+n}$
8. Two events A and B are independent if and only if
   $P(A/B) = P(A)$
   $P(B/A) = P(B)$
9. If A and B are independent event associated with random experiment, then
   i) $\overline{A}, B$
   ii) $\overline{B}, A$
   iii) $\overline{A}, \overline{B}$ are also independent event.

Coin Toss Probability

1. A coin is tossed twice at random. What is the probability of getting

(i) at least one head (ii) the same face?

a)i)1/5 & 2/5 b)i) ½ & 2/6
c)i)3/4 & ½ d)i) 1/3 & 4/7

2. If three fair coins are tossed randomly 175 times and it is found that three heads appeared 21 times, two heads appeared 56 times, one head appeared 63 times and zero head appeared 35 times. What is

the probability of getting when 3 coins are tossed randomly, the only possible outcomes?

a) 2 **b) 1**
c) 3 d) 4

3. Two coins are tossed randomly 120 times and it is found that two tails appeared 60 times, one tail appeared 48 times and no tail appeared 12 times. what is the probability of getting while tossing 2 coins simultaneously, the only possible outcomes?

a) 2 **b) 1**
c) 3 d) 4

EXAMPLES

Q. 1. From a well shuffled pack of 52 playing card, one card is drawn. Find the probability of getting
i) Queen
ii) black and queen
iii) a face card

a)i) 1/13 b)i) 1/12
c) i)1/15 d)i) 1/20

a) ii) 1/26 b) ii) 1/25
c) ii) 1/27 d) ii) 1/30

a) iii) 4/13 b) iii) 4/14
c) iii) 3/12 d) iii) 5/23

Q.2 Two dices are thrown simultaneously. What probability of obtaining a total score of seven?

a)5/6 **b)1/6**
c)1/2 d) 1/3

Q.3. Two coins are tossed. What is the probability of getting
i) at least two head
ii) at most two head

a)i)1/5 b)i) ½
c)i)1/4 d)i) 1/3
a) ii) 2 b) ii) 3
c) ii) 1 d) ii) 5

Q.4. What is the chances that a leap year, selected at random, will contain 53 cards Sundays ?

a)2/7 b) 2/8
c) ½ d) 1/3

Q.5. A committee of 5 persons is to be constituted from a group of 6 gents and 8 ladies. If the selection is made randomly, find the probability that are 3 ladies and 2 gents in the committee.

a) 50/120 **b) 60/143**
c) 65/143 d) 50 /143

Q. 6. One bag A contains 4 white balls and 2 black balls; another bag B contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that one is white and one is blac

a) 15/24 b) 15/13
c) 16/24 **d) 13/24**

Q.7.A box contains 10 bulbs, out which just three are defective if a random sample of five bulbs is drawn, find the probability that the sample contains
i) exactly one defective
ii) exactly two defective bulbs
iii) no defective bulbs

a)i) 5/12 b)i)3/12
c) i)4/12 d)i) 6/12

ii)5/12 ii) 3/12
ii) 4/12 ii) 6/12

iii)1/12 iii)2/12
iii) 3/12 iii) 4/12

Q.8. Five persons entered the left cabin on the ground floor of an 8 floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with first. Find out the probability of all five persons leaving at different floor.

a) 360/ 2501 b) 560/2560
c) 350/2460 d) 340/3450

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Q.9. The odds in favor of an event A are 3 : 4. The odds against another independent event B are 7 : 4. What is the probability that at least one of the events will happen?

a)6/11 **b) 7/11**
c) 8/11 d)none

Q.10. there are 300 orphans in Bal Sudhar Ghar of which 15 are handicapped, 13 are mentally unstable. If an orphan is selected at random, what is the probability that he is neither handicapped nor mentally disable?

a) 13/12 b) 14/12
c) 11/12 d) 15/12

Q.11.An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and the fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane ?

a) 0. 6789 b) 0. 6578
c) 0.6567 **d) 0. 6876**

Q. 12. A can solve 90% of the problem given in a book and B can solve 70%. What is the probability that at least one of them will solve the problem, selected at ramdom from the book?

a) 0.97 b) 0.98
c) 0.89 d) 0.95

Q.13. a speaks truth in 75% of cases and b in 80% of cases. In what percent of cases are they likely to contradict each other in narrating the same incident ?

a) 45/100 **b) 35/100**
c) 55/100 d) 65/100

Q.14. The probability of A, B, C solving a problem are 1/3, 2/7 and 3/8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve it.

a) 35/56 b) 45/56
c) 25/56 d) 55/56

Q.15. A and B toss a coin alternately with one of them tossed a head and wins the game. If A starts the game, find their respective probabilities of winning.

a) ½ b)1/4
c) 1/5 **d) 1/3**

PRACTICE EXERCISE

1.Three unbiased coins are tossed. The probability of getting two heads is

a)$\frac{1}{4}$ *b)* $\frac{1}{8}$ ***c)*** $\frac{3}{8}$ *d)* 1

2. Two dices are thrown simultaneously. The probability of obtaining a total score of 8 is

a)$\frac{5}{36}$ b)$\frac{1}{6}$ c)$\frac{1}{9}$
d)$\frac{7}{36}$

3. Three coins are tossed simultaneously. The probability of getting at most 2 heads is

a) $\frac{3}{8}$ b) $\frac{1}{6}$ c) $\frac{1}{9}$ **d)** $\frac{7}{8}$

4.A pair of dice are thrown. The probability of getting the sum as a multiple of 3 is

a) $\frac{1}{3}$ b) $\frac{5}{12}$ c) $\frac{1}{6}$ d) $\frac{11}{36}$

5.A student has to select 3 subjects out of 6 subjects Mathematics, Biology, Chemistry, Physics, English and Computer Science. If he has chosen Mathematics. What is the probability of Biology being chosen?

a) 2/3 **b) 2/5**
c) 1/3 d) 3/5

6.A box has 10 bulbs, out of which 4 are defective. If 3 bulbs are choosen at random, then the probability that none of that one is red and other white is

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a) 1/6 b) 2/3
c) 8/11 d) 5/22

7.A bag contains 4 white and 6 red balls. Two draws of one ball each are made without replacement. The probability that one is red and other white is ?
a) 10/15 b) 6/9
c) 8/15 d) 4/9

8.Three persons A, B, C are in the car race. A is twice as likely to win as B and B is twice as likely to win as C. What is the probability for A to win the race ?
a) 1/7 b) 2/7
c) 4/7 d) none

9.If the probability of A to fail in examination is 0.2 and that of B is 0.3, then the probability that either A or B fails is
a) 0.1 **b) 0.44**
c) 0.06 d) 0.5

10.an elevator starts with 5 passengers and stops at 8 different floors of the house. The probability of all the 5 passengers alighting at different floor
a) 105/512 b) 137/228
c) 99/512 d) 107/512

11.A bag contains 6 red, 4 white and blue balls. If three balls are drawn one by one without replacement. The probability of getting all three balls red is
a) 4/5 b) 18/91
c) 15/78 **d) 4/17**

12. A bag contains 4 red, 3 blue and 2 white balls. If three balls are drawn one by one without replacement. The probability of getting all three balls red is
a) 2/9 b) 1/3
c) 2/7 **d) 1/21**

13. A dice is thrown . The probability that the number on the dice is not divisible by 3 is
a) 2/3 b) 1/3
c) 1/6 d) 5/6

14. Four cards are drawn at random from a pack of 52 playing cards. The probability of getting all the four cards of the same suit is
a) 1/20825 b) 2197/20825
c) 44/4165 d) 92/833

15. Four cards are drawn at random from a pack of 52 cards. The probability of getting all the four cards of the same number is
a) 325/833 **b) 1/20825**
c) 44/4165 d) 92/833

16. A box contains 20 machine parts, 5 of them being standard. A worker takes out 3 parts at random. Find the probability that at least 1 of the 3 turns out to be standard
a) 135/1467 b) 62/228
c) 137/228 d) none

17.In a track contest, the odds that X will win the race are 1 : 2 and the odds that Y will win race are 2 : 3. The probability that X or y wins the race is
a) 11/15 b) 9/17

c) 3/13 d) ¾

18. A problem in mathematics is given to 3 students whose chances of solving it are ½ , 1/3, ¼. The probability that the problem is solved is
a) ¼ b) ½
c) 5/6 **d) ¾**

19. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. The probability that the problem will be solved is
a) 1/21 **b) 16/21**
c) 8/21 d) 11/21

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20. There are two men aged 26 and 30 years. The probability to live 35 years more is 0.67 for 26 years old man and 0.60 for 30 years old man. The probability that at least one of these men will be alive 35 years hence is
a) 0.44 b) 0.6976
c) 0.868 d) 0.866

21. The probabilities of three mutually exclusive events are P(A) = 2/3, P(B) = ¼ and P(C)= 1/6. The statement is
a) true **b) wrong**
c) not meaningful d) can not be decided

22. A bag contains 7 red and 5 white balls. Two successive draws of 2 balls are made without replacement. The probability that the in the first draw 2 white balls and in the second draw 2 red balls be choosen is
a) 14/33 **b) 7/99**
c) 37/45 d) 13/24

23. A speaks truth in 65% of the cases and B in 80 of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact
a) 41% b) 44%
c) 35% d) 48%

24. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occur is 1/3. The probability of occurrence of A is
a) ½ or 1/3 b) 2/3
c) ¼ d) 5/6

25. In a certain school, 20% students failed in English , 155 students failed in Mathematics and 10% students are failed in both English and Mathematics. A student is selected at random. If he failed in English, what is the probability that he also failed in Mathematics?
a) ¼ b) ¾
c) ½ d) 3/8

26. The probabilities that on a certain the two trains Chennai and Ahmedabad arrive on time at Mumbai are 0.93 and 0.89. the probability that both the trains arrive at time is 0.87. The probability that at least one train on time is
a) 0.80 b) 0.82
c) 0.85 **d) 0.95**

27. A can hit a target 4 times in 5 shots, B hits 3 times in 4 shots and C hits twice in 3 shots. They fire together. The probability that at least two shots in the target is
a) 13/30 b) 1/27
c) 5/6 d)3/4

28. Probability of occurrence of two events A and B are 0.25 and 0.50. Probability of simultaneous occurrence of A and B are 0.14. The probability that neither A nor B occr is
a) 0.28 **b) 0.39**
c) 0.47 d) 0.61

29. A bag contains 4 red and 7 green balls and another bag contains 8 red and 5 green balls. One ball is transferred from first bag to the second bag without noticing its color. A ball is then drawn from the second bag. The probability that it is green ball is
a) 31/77 b) 31/81
c) 59/81 d) 42/77

30. A student appears for test A, B and C. A student is successful if he passes either in test A, B or A, c. The probability of the student to be successful is ½ then
a) p = q = 1 b) p=q=1/2
c) p = 1, q = 1 d) p= 1, q= ½

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