

PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULA

Factorial Notation: Let n be a positive integer. Then factorial n, denoted by n! is defined as:

$$n! = n(n-1)(n-2) \dots 3, 2, 1.$$

Examples: i) $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$

$$\text{ii) } 4! = (4 \times 3 \times 2 \times 1) = 24 \text{ etc.}$$

We define $0! = 1$.

Permutations: The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb)

Ex 2. All permutations made with the letters a, b, c taking all at a time are (abc, acb, bac, bca, cab, cba)

Number of permutations : Number of permutations of n things, taken r at a time, is given by :

$${}^n P_r = \frac{n(n-1)(n-2) \dots (n-r+1)}{(n-r)!} = \frac{n!}{(n-r)!}$$

Examples :

$$\text{(i) } {}^6 P_2 = (6 \times 5) = 30.$$

$$\text{(ii) } {}^7 P_3 = (7 \times 6 \times 5) = 210.$$

Cor. Number of all permutations of n things, taken all at a time = n!

An Important Result : If there are n objects of which p₁ are alike of one kind; p₂ are alike of another kind; p₃ are alike of third kind and so on and P_r are alike of rth kind, such that (P₁ + P₂ + 1 +P_r) = n.

Then, number of permutations of these n objects is :

$$\frac{n!}{(P_1!) \cdot (P_2!) \dots (P_r!)}$$

Combinations: Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex.1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note that AB and BA represent the same selection.

Ex.2. All the combinations formed by a, b, c taking two at a time are ab, bc, ca.

Ex.3. The only combination that can be formed of three letters a, b, c taken all at a time is abc.

Ex. 4. Various groups of 2 out of four persons A, B, C, D are: AB, AC, AD, BC, BD, CD.

Ex 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of combinations: The number of all combinations of n things, taken r at a time is :

$${}^n C_r = \frac{n!}{(r!(n-r)!)} = \frac{n(n-1)(n-2) \dots \text{to } r \text{ factors}}{r!}$$

Note that : ${}^n C_n = 1$ and ${}^n C_0 = 1$.

An Important result : ${}^n C_r = {}^n C_{(n-r)}$.

Example :

$$\text{(i) } {}^{11} C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330$$

$$\text{(ii) } {}^{16} C_{13} = {}^{16} C_{(16-13)} = {}^{16} C_3 = \frac{(16 \times 15 \times 14)}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$$

Concept Question

Ex.1. Evaluate : $\frac{30!}{28!}$

- a) 800
- b) 850
- c) 870
- d) 860

2. Find the value of

- (i) ${}^{60} P_3$
- (ii) ${}^4 P_4$
- a) 205320, 24
- b) 267057, 20
- c) 256575, 24
- d) 179287, 15

3. Find the value of

- (i) ${}^{10} C_3$
- (ii) ${}^{100} C_{98}$
- (iii) ${}^{50} C_{50}$
- a) 150, 4500, 2
- b) 120, 4950, 1
- c) 150, 2356, 4
- d) 130, 4679, 3

4. How many words can be formed by using all letters of the word 'BIHAR' ?

- a) 150
- b) 140
- c) 156
- d) 120

5. How many words can be formed by using all letters of the word 'Chennai' ?

- a) 5040 **b) 2520**
c) 3028 d) 120

6. How many words can be formed by using all letters of the word 'Apple' ?

- a) 60** b) 40
c) 56 d) 120

7. How many words can be formed by using all letters of the word 'Leader' ?

- a) 150 b) 140
c) 156 **d) 360**

8. How many words can be formed by using all letters of the word 'Machine' ?

- a) 1506 b) 1402
c) 1056 **d) 5040**

9. How many words can be formed by using all letters of the word 'Engineering' ?

- a) 277300 b) 227700
c) 277200 d) None of these

10. How many words can be formed by using all letters of the word 'Mississippi' ?

- a) 34650** b) 34560
c) 34300 d) None of these

11. How many words can be formed by using all the letters of the word 'DAUGHTER' so that the vowel always come together?

- a) 4320** b) 4500
c) 4400 d) 6000

12. How many words can be formed by using all the letters of the word 'CORPORATION' so that the vowel always come together?

- a) 50400** b) 40500
c) 54400 d) 65000

13. How many words can be formed by using all the letters of the word 'ENGINEERING' so that the vowel always come together?

- a) 5200 b) 4500
c) 4200 d) 5000

14. How many words can be formed by using all the letters of the word 'MISSISSIPPI' so that the vowel always come together?

- a) 420 b) 450
c) 820 **d) 840**

15. How Many words can be formed from the letters of the word 'DIRECTOR' so that the vowel are always together ?

- a) 2180 b) 2400
c) 2380 **d) 2160**

16. How many words can be formed from the letters of the word 'EXTRA', so that the vowels are never together ?

- a) 130 **b) 120**
c) 150 d) 140

17. How many words can be formed from the letters of the word 'ENGINEERING', so that the vowels are never together?

- a) 27300** b) 27200
c) 27400 d) 23700

18. How many words can be formed from the letters of the word 'CORPORATION', so that the vowels are never together?

- a) 3326400 **b) 327600**
c) 327400 d) 423700

19. In how many ways can a cricket eleven be chosen out of a batch of 15 players?

- a) 1365** b) 1323
c) 1453 d) 1546

20. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

- a) 400 b) 500
c) 200 d) 300

21. From a group of 7 men and 6 women five persons are to be selected to form a committee so that at least

3 men are three on the committee. In how many ways can it be done?

- a) 564 b) 645 c) 735
d) **756** e) None of these

22. In a group of 6 boys and 4 girls, four children are to be selected. In how many ways can they be selected such that at least one boy should be there?

- a) 159 b) 194 c) 205
d) **209** e) None of these

23. In how many different ways can the letters of the word 'CABINET' be arranged so that the vowels may occupy only the odd positions ?

- a) 210 b) **576** c) 144
d) 60 e) 3456

2. How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'APPRECIATE', if repetition of letters is not allowed?

- a) 40 b) 400
c) **5040** d) 2520

PRACTICE EXERCISE

1. The value of ${}^{75}P_2$ is :

- a) 2775 b) 150
c) **5550** d) None of these

2. How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

- a) 40 b) 400
c) **5040** d) 2520

3. How many words with or without meaning, can be formed by using all the letters of the word, 'DELHI', using each letter exactly once?

- a) 10 b) 25
c) 60 d) **120**

4. In how many ways can the letters of the word 'APPLE' be arranged?

- a) 720 b) 120
c) **60** d) 180

5) In how many ways can the letters of the word 'LEADER' can be arranged ?

- a) 72 b) 144
c) **360** d) 720 e) None of these

6. In how many different ways can the letters of the word 'RUMOUR' be arranged?

- a) **180** b) 90 c) 30
d) 720 e) None of these

7. How many words can be formed by using all the letters of the word, 'ALLAHABAD'?

- a) 3780 b) 1890 c) **7560**
d) 2520 e) None of these

8. How many arrangements can be made out of the letters of the word 'ENGINEERING' ?

- a) **720** b) 1440 c) 2880
d) 3600 e) 17280

9. How many words can be formed from the letters of the word 'SIGNATURE' so that the vowels always come together ?

- a) 720 b) 1440 c) 288
d) 3600 e) **None of these**

10. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together ?

- a) 120 b) 720 c) **4320**
d) 2160 e) None of these

11. In how many different ways can the letters of the word 'SOFTWARE' be arranged in such a way that the vowels always come together?

- a) 120 b) 360 c) 1440
d) 13440 e) **720**

12. In how many different ways can the letters of the words 'LEADING' be arranged in such a way that the vowels always come together ?

- a) 360 b) 480 **c) 720**
d) 5040 e) None of these

13. In how many different ways can the letters of the word 'JUDGE' be arranged in such a way that the vowels always come together?

- a) 48** b) 120
c) 124 d) 160

14. In how many different ways can the letters of the word 'AUCTION' be arranged in such a way that the vowels always come together ?

- a) 30 b) 48 c) 144
d) 576 e) None of these

15. In how many different ways can the letters of the word 'BANKING' be arranged so that the vowels always come together ?

- a) 120 b) 240 c) 360
d) 540 **e) None of these**

16. In how many ways can the letters of the word 'CORPORATION' be arranged so that vowels always come together?

- a) 810 b) 1440 c) 2880
d) 50400 e) 5760

17. In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that vowels always come together ?

- a) 10080 b) 4989600
c) 120960 d) None of these

18. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions ?

- a) 32 b) 48 **c) 36**
d) 60 e) 120

19. In how many different ways can the letters of the word 'MACHINE' be arranged so that the vowels may occupy only the odd positions ?

- a) 210 **b) 576** c) 144
d) 60 e) 3456

20. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women ?

- a) 63** b) 90 c) 126
d) 45 e) 135

21. In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women ?

- a) 266 b) 5040 **c) 11760**
d) 86400 e) none of these

22. From a group of 7 men and 6 women five persons are to be selected to form a committee so that at least 3 men are three on the committee. In how many ways can it be done ?

- a) 564 b) 645 c) 735
d) 756 e) None of these

23. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there ?

- a) 159 b) 194 c) 205
d) 209 e) None of these

24. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways an 3 balls be drawn from the box, it at least one black ball is to be included in the draw ?

- a) 32 b) 48 **c) 64**
d) 96 e) None of these

25. How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated ?

- a) 5 b) 1540 c) 199
d) 3672 e) None of these

26. In how many ways can 21 books on English and 19 books Hindi be placed in a row on a shelf so that two books on Hindi may not be together ?

- a) 3990 **b) 1050** c) 25200
d) 21400 e) None of these

27. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

- a) 210 b) 1050 c) **25200**
d) 21400 e) None of these